STEP MATHEMATICS 1

2018

Hints and Solutions

Introductory Remarks

This document should be read in conjunction with the corresponding mark scheme in order to gain full benefit from it. Since the complete solutions appear elsewhere, much of this *Hints and Solutions* document will concentrate more on the "whys and wherefores" of the solution approach to each question and less on the technical details.

The solutions that follow, presented either in outline or in full, are by no means the only ones, not even necessarily the 'best' ones. They are simply intended to be the ones that, on the evidence of the marking process, appear to be the ones which arose most frequently from the ideas produced by the candidates and that worked for those who could force them through to a conclusion. If you "see" things in a different way, I hope you can still both follow and appreciate what is given here.

Question 1

The first question on the first STEP is always intended to have a more familiar feel to it and this is one such question.

Equating the two expressions for y, for the two given entities to meet, then cancelling the obvious x, gives a simple quadratic: and you should know of a few ways to tackle it to find the coordinates (don't forget the y-coordinate) of both P and Q. The first "curve" is a line through the origin and the second is a cubic, also through O. The repeated factor in $y = x(b - x)^2$ indicates tangency at $x = b$ so the curve – a "positive" cubic, in the sense of the sign of the dominant (x^3) term – has the classical up-down-up shape, passes through O and touches the x-axis at b.

Next, "equation of tangent" should suggest that you to differentiate, substitute in the values of x and y at P, and build up the tangent's equation in the form required. Remember to *show clearly* how the answer is obtained; a lot of candidates lose marks by jumping to the given answer without justifying fully their arrival. A given answer always requires more detailed working; even if you are perfectly capable of doing all the working 'in your head', the working must be shown in writing.

The areas in the next part of the question can be found by integration but it is important to have a decently drawn diagram to point you in the right direction; especially as one of the regions in question is a triangle, which hardly requires the use of calculus (although, on this occasion, incorporating all into a single integral is just as straightforward).

Finally, now that you have algebraic expressions for S and T, you are asked to establish an inequality, $S > \frac{1}{3}T$. Inequalities can often be quite difficult to handle well. However, there is a sensible "trick" that often works well, and that is to prove an equivalent inequality involving zero: in this case, either $S - \frac{1}{3}T > 0$ or $3S - T > 0$. Establishing the sign of a single expression (such as $3S - T$) is almost always far more easily accomplished than having to compare a variable LHS with a variable RHS.

Question 2

In this question, the important thing is to be careful with the directions of the inequalities, and particularly the sign of anything that you intend to multiply or divide by (variable items are always tricky as they can be both positive and negative … at different "times", of course). Remember to say why you are doing what you are doing clearly; the biggest loss of marks in these sorts of questions invariably comes from those who write down statements, often correct ones, but the marker is unable to spot why they have been written down. Referring "invisibly" to a result a dozen lines up the page is not generally sufficient, and the markers are not required to hunt around and identify the reasons why you might have made such-and-such a conclusion and give you credit for their own understanding of the written work confronting them: spell it out clearly for them. If necessary, give each key result a circled number reference so you can cite them later on: see part **(iii) below** for an example.

In this question, to begin with, you are reminded of a result commonly referred to as the *Change-Of-Base-Formula*. Establishing it is easy provided you realise that the statements $x = \log_b c$ and $b^x = c$ are equivalent. Taking logs to base a for this second statement then gives the opening result.

In **(i)**, all that is needed is to take logs to base 10 with both the given (useable) statement $\pi^2 < 10$ and then with the LHS of the given inequality (using the *COBF* noted above). This gives (essentially) just the two lines of working: $\pi^2 < 10 \Rightarrow \log_{10}\pi^2 < 1 \Rightarrow 2\log_{10}\pi < 1 \Rightarrow \log_{10}\pi < \frac{1}{2}$ ① $\Rightarrow \dfrac{1}{\log_{10}\pi} > 2$

and $\dfrac{1}{\log_2\pi} + \dfrac{1}{\log_5\pi} = \dfrac{\log_{10}2}{\log_{10}\pi} + \dfrac{\log_{10}5}{\log_{10}\pi}$ by the COBF $= \dfrac{\log_{10}2 + \log_{10}5}{\log_{10}\pi} = \dfrac{\log_{10}10}{\log_{10}\pi} = \dfrac{1}{\log_{10}\pi} > 2$.

Part **(ii)** is a little more protracted, and this time the COBF needs to use logs to base e (i.e. ln's).

$\log_2\dfrac{\pi}{\mathrm{e}} > \frac{1}{5} \Rightarrow \log_2\pi - \log_2\mathrm{e} > \frac{1}{5} \Rightarrow \dfrac{\ln\pi}{\ln 2} - \dfrac{\ln\mathrm{e}}{\ln 2} > \frac{1}{5}$ by the COBF $\Rightarrow \ln\pi > 1 + \frac{1}{5}\ln 2$

and $\mathrm{e}^2 < 8 \Rightarrow \ln\mathrm{e}^2 < \ln 2^3 \Rightarrow 2\ln\mathrm{e} < 3\ln 2 \Rightarrow \ln 2 > \frac{2}{3}$

so that, putting the two together, $\ln\pi > 1 + \frac{1}{5}\ln 2 > 1 + \frac{1}{5}.\frac{2}{3} = \frac{17}{15}$, as required.

Notice the way in which it is easiest to keep the direction of the inequalities consistent. This is a technique we shall continue to adopt in **(iii)**. Taking logs to base 10 (suggested by the presence of $\log_{10}2$), we have

$20 < \mathrm{e}^3 \Rightarrow \log_{10}20 < 3\log_{10}\mathrm{e} \Rightarrow \log_{10}10 + \log_{10}2 < 3\log_{10}\mathrm{e} \Rightarrow 1 + \log_{10}2 < 3\log_{10}\mathrm{e}$ ②

and $\pi^2 < 10 \Rightarrow \log_{10}\pi < \frac{1}{2}$ from ① above

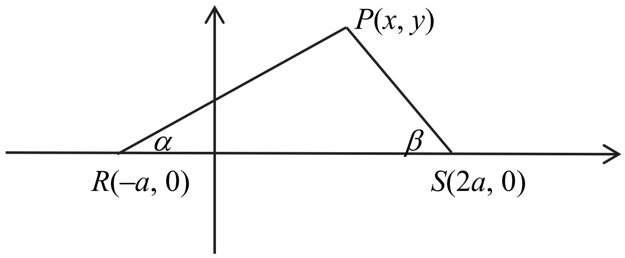
and $\frac{3}{10} < \log_{10}2$ ③

It follows that $\frac{13}{10} < 1 + \log_{10}2 < 3\log_{10}\mathrm{e}$ using ③ and ② so that $\frac{13}{30} < \log_{10}\mathrm{e} = \dfrac{\ln\mathrm{e}}{\ln 10} = \dfrac{1}{\ln 10} \Rightarrow \ln 10 < \frac{30}{13}$

and $\log_{10}\pi < \frac{1}{2}$ (①) $\Rightarrow \dfrac{\ln\pi}{\ln 10} < \frac{1}{2} \Rightarrow \ln\pi < \frac{1}{2}\ln 10 < \frac{1}{2}.\frac{30}{13} = \frac{15}{13}$.

Question 3

This question involves a collection of fairly simple ideas, especially in part **(i)**, but leaves the candidate to explore the difference between "$\Rightarrow$" and "$\Leftarrow$" lines of reasoning with part **(ii)**. Again, at the outset, a good diagram can help in getting started:



Noting first that $\tan\alpha = \dfrac{y}{x+a}$ and $\tan\beta = \dfrac{y}{2a-x}$, if $\beta = 2\alpha$ then $\tan 2\alpha = \dfrac{y}{2a-x} = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\,\frac{y}{x+a}}{1-\frac{y^2}{(x+a)^2}}$.

Since $y \neq 0$, this all tidies up to the required $y^2 = 3\left(x^2 - a^2\right)$.

In **(ii)**, starting from $y^2 = 3\left(x^2 - a^2\right)$ leads backwards to $\tan\beta = \tan 2\alpha$. The difference now is that this gives the more general result, when "un-trigging" (think about your quadrants work, or the symmetries and periodicities of the tan function), that $\beta = 2\alpha + n\pi$. All that is needed now is to consider which integer values of n give a viable triangle setting for α and β (one of which must be the obvious $\beta = 2\alpha$, of course.)

Question 4

Most of the difficulty inherent in this question is of a technical nature. It should be clear that you must differentiate and show that $(1 - \ln x)$ is a factor of both $\mathrm{f}(x)$ *and* $\mathrm{f}'(x)$, but finding the correct derivative will usually involve application of the product, quotient *and* chain rules.

In **(i)**, to begin with, the limits are an irrelevance, since the integrand is the same in both portions of the function F. Most of the time, in standard A-level papers, you are given the required substitution; not so here, although the only really obvious simplifying choice of $u = \ln t$ turns out to 'do the trick' perfectly well. The other thing that then turns out to be different from A-level is that, here, it is really important to integrate t^{-1} as $\ln|t|$; however, there is a big push to consider this when you realise that, in the two explicitly stated regions of $0 < x < 1$ and $x > 1$, the ln function is first negative and then positive. Since all powers of $\ln t$ are even, it turns out that

$$\mathrm{F}(x) = \ln|\ln x| - (\ln x)^2 + \tfrac{1}{4}(\ln x)^4 + \tfrac{3}{4}$$

in each of the two regions. Indeed, the even powers, and the modulus, ensure that $\mathrm{F} = \mathrm{F}^{-1}$.

The sketch of the curve is interesting but all the clues are there already. There is a vertical asymptote of $x = 1$ (since $\ln(\ln 1) = \ln 0$ is inadmissible; thereafter, the final result of part **(i)** actually tells you that the curve exhibits a sort of reflection symmetry between the two regions. The final key observation is that each region has a point of inflexion as F crosses the x-axis.

Question 5

In many ways, as with a lot of STEP questions, the real difficulty here lies in having to put together, without a great deal (if any) of notification, a number of different mathematical ideas from almost anything you may have learnt over the years, but to do so in a suitably sophisticated way.

For a starter, you are asked for the *most general* quartic that leaves a remainder of 1 upon division by … . The required polynomial is clearly $k(x-1)(x-2)(x-3)(x-4)+1$ (the most common error being to think the leading coefficient is always 1) and it is helpful to note that k cannot be zero. This leads to the suggested $P(x) = k(x-1)(x-2)(x-3) \ldots (x-N) + 1$, $k \neq 0$, in **(ii)** and substituting $x = N + 1$ leads to the required conclusion that $P(N+1) \neq 1$ either by brute force or by a 'contradiction' argument that notes that, if this *were* the case, then we would have a polynomial of degree N with $(N+1)$ distinct roots.

For the very last part of **(ii)**, notice that you are only required to find one such r and, after a bit algebra, it becomes clear that $r = 2$ does the job; verifying that it does so is rather easy.

The same sorts of ideas then come into play in part **(iii)**, where we have a quartic polynomial equation, $S(x) - 1 = 0$, with four distinct integer zeroes a, b, c and d. Setting $x = e$ then gives

$$(e-a)(e-b)(e-c)(e-d) = 17$$

and, although 17 *does* have four distinct integer factors (± 1, ± 17), the two 17s cannot both be used. In **(b)** a similar consideration of $S(0)$ leads to $abcd = 16$ so that only the numbers ± 1, ± 2, ± 4, ± 8, ± 16 can be used. There are several possible approaches (three of which appear in the published Mark Scheme) but the most important things needed to be clear and systematic in your presentation.

Question 6

This does *look* horrible (potentially, at least) and, in fact, it can indeed be solved in a very lengthy way if one is not careful. The opening result is a simple application of the well-known "Addition Formulae" for $\cos(A \pm B)$. Expanding the LHS of the given result so that each term on the LHS is of the form $2\sin P \sin Q$ leads to a "collapsing" (or "telescoping") series where almost all of the terms appear once negatively and once positively, leaving only the first and last ones standing.

This result is then used in **(i)**, once you have correctly split the area into n equal rectangular strips of equal bases but heights given by the height of each strip's midpoint: choosing the value of θ suitably and collapsing the series gives the required outcome, $A_n = \dfrac{\pi}{n}\operatorname{cosec}\dfrac{\pi}{2n}$, which I have written in this form for later use.

The *Trapezium Rule* formula gives a similar result once you have factored in (to both sides, of course) the $\sin\dfrac{\pi}{2n}$, although a satisfactorily simplified answer requires the use of a final step involving $\cos(\pi - \theta) = -\cos\theta$ in order to end up with $B_n = \dfrac{\pi}{n}\cot\dfrac{\pi}{2n}$.

The two results of part **(iii)** now turn out to involve little more than bits of (unprompted) trig. identity work on the expressions for A_n and B_n in the above forms.

Question 7

To begin with, this is relatively straightforward: setting $x = \frac{pz+q}{z+1}$ in $x^3 - 3pqx + pq(p + q) = 0$ gives an expression which can be factorised into $(p - q)^2(pz^3 + q) = 0$ and, since $p \neq q$, it follows that $a = p$ and $b = q$.

In **(ii)**, we require $pq = c$ and $p + q = \frac{d}{c}$, and we can then consider p and q as the roots of the quadratic equation $y^2 - \frac{d}{c}y + c = 0$, which has two, real distinct roots provided its discriminant is positive.

Part **(iii)** is clearly going to be of the required form (but it is still important to demonstrate that this is so). It should be obvious, with a moment's thought, that there are going to appear to be two outcomes, but also that they will turn out to give the same result, since p and q are interchangeable. In the simpler version, we get $p = -1$, $q = 2$, which leads to $2 - z^3 = 0$ and $z = \sqrt[3]{2}$.

In **(iv)**, it is routine to find a linear factor (by using the factor theorem) and $x = p$ can be seen to give a zero of the given cubic expression. Proceeding as usual, one obtains $(x - p)^2(x + 2p) = 0$ and $x = p$ or $-2p$. These, in turn, yield the required roots of $x^3 - 3cx + d = 0$ in the case when $d^2 = 4c^3$.

Question 8

What is especially pleasing about this question (from a teaching point of view) is that it principally deals with the simple details of calculus – both differentiation and integration – unencumbered by the specifics of individual functions (even though the “s” and “c” do look as if they have *something* to do with sine and cosine). In this sense, there is something fundamentally “pure” about the application of the various rules that are being effected.

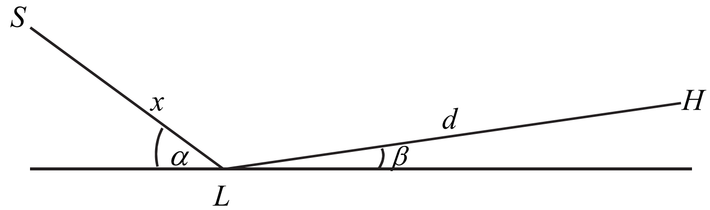
In **(i)**, one must simply appreciate that a function differentiating to zero is constant and, in which case, whatever value one substitutes in, that constant will be the output. The required results in **(ii)** follow from the use of the product and quotient rules and the identity obtained in **(i)**.

The first result of **(iii)** follows directly from the given statement $c'(x) = -s(x)^2$, requiring only the most basic grasp of integration as anti-differentiation. For the second result, candidates need to identify how to split what appears to be the single term, needing to be integrated, into two useable “parts” and, following a bit of algebra, recognising a “reverse chain rule” integration (which, of course, can always be done with a suitable substitution).

In a similar vein, part **(iv)** tests your capacity to think in terms of the key elements of the substitution –integration process; while **(v)** then requires you to deploy a mixture of all of the above skills.

Question 9

A simple diagram can be remarkably effective in enabling you to put your thoughts in order, even if it consists of little more than this:



Now, in order to reach H from S, the GPE lost from S to L must exceed that gained from L to H (so that there is a non-negative amount of energy left for a non-negative kinetic energy). Thus

$$mg\, x \sin\alpha \geq mg\, d \sin\beta \quad \text{and} \quad x \sin\alpha \geq d \sin\beta\,.$$

The rest of the background work for the question can be done using the so-called “constant acceleration formulae”,

$$s = ut + \tfrac{1}{2}at^2 \text{ ❶},\ \ v = u + at \text{ ❷},\ \ v^2 = u^2 + 2as \text{ ❸} \ \text{ and } \ s = \tfrac{1}{2}(u + v)t \text{ ❹},$$

with suitably chosen values of the variables.

Using ❸ $v^2 = 2gx \sin\alpha \Rightarrow v = \sqrt{2gx\sin\alpha}$, and ❹ then gives $t_1 = \dfrac{2x}{v} = \sqrt{\dfrac{2x}{g\sin\alpha}}$.

Next, ❶ gives $d = vt_2 - \frac{1}{2}g \sin\beta\ t_2^2$, which is a quadratic equation in t_2 (and it is the first, smaller, root which is required). The bulk of the working for this main part of the question is then taken up in re-forming $T = t_1 + t_2$ into the given answer,

$$\left(\frac{g\sin\alpha}{2}\right)^{\frac{1}{2}} T = (1+k)\sqrt{x} - \sqrt{k^2x - kd}\,.$$

Once this has been obtained, we are required to do the usual sorts of calculus:

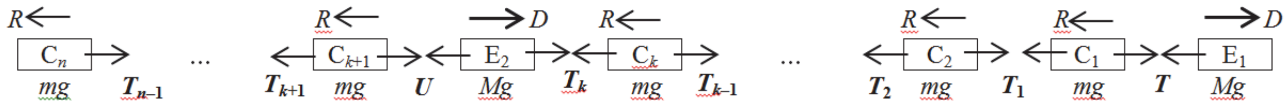
$$\frac{\mathrm{d}}{\mathrm{d}x}(RHS) = \frac{1+k}{2\sqrt{x}} - \frac{k^2}{2\sqrt{k^2x-kd}} = 0 \ \text{ when } \ \frac{(1+k)^2}{x} = \frac{k^4}{k^2x-kd}$$

$$\Rightarrow \left(1+2k+k^2\right)\left(kx-d\right) = k^3x \Rightarrow \left(k+2k^2+k^3-k^3\right)x = (1+k)^2 d$$

and $x = \dfrac{(1+k)^2}{k(1+2k)}d$.

Question 10

A lot of mechanics questions are best approached with a helpful, carefully marked, diagram:



The rest of the question involves the application of N2L (*Newton's Second Law*) to various bits of this system, or the whole thing. The trick is to let what is asked for guide your thoughts to the best "bit" to choose to apply it to.

To begin with, apply N2L to the whole train: $2D - nR = (2M + nm)a$ and then to the first engine (E_1): $D - T = Ma$. Eliminating D gives the required result for **(i)**.

The next result may call for a bit of investigation, but each application of N2L takes time, so it is best to look at a representative carriage between E_1 and E_2 first, and then at one after E_2 (if necessary). The first of these gives $T_{r-1} - T_r = R + ma$, for $1 \le r \le k$, which is positive, so the tensions are decreasing. Then, the same principle will also apply to the carriages after E_2.

Next, use N2L for the final section of the train: $U - (n - k)R = (n - k)ma$.

Thereafter, we examine the quantity $T - U$, eliminate the D in the numerator of **(i)**'s expression for T, and end up with

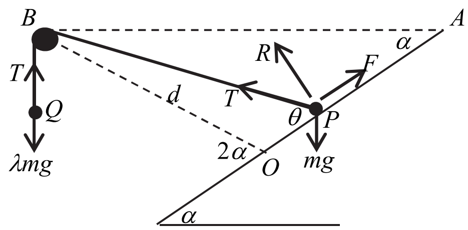
$$T - U = (k - \tfrac{1}{2}n)(R + ma)$$

which gives the required result in **(ii)** that $T > U$ provided $k > \frac{1}{2}n$.

Having seen that all the tensions in the couplings between the components in the front part of the train are positive, **if** the suggested outcome does occur, it must be in one of the carriages in the back part of the train. Try N2L for E_2: $D + T_k - U = Ma \Rightarrow T_k = U + Ma - D = U + Ma - (T + Ma) = U - T < 0$ from the result of **(ii)**.

Question 11

If the diagrams for the other mechanics questions were of questionable value, in questions such as this they are an absolute necessity:



In **(i)**, we can reason as follows: as $P \to O$, $\theta \to 2\alpha$ and as $P \to A$, $\theta \to \alpha$ so $\alpha \le \theta \le 2\alpha$.

In **(ii)**, the standard process of *resolution of forces* comes into play. These give

Res. ‖ plane for P:	$mg\sin\alpha + T\cos\theta = F$	❶
Res. $\perp^r$. plane for P:	$R + T\sin\theta = mg\cos\alpha$	❷
Res. ↑ for Q:	$T = \lambda mg$	❸
Friction Law (eqlm.):	$F \le \mu R$	❹

and it follows that $R = mg\cos\alpha - T\sin\theta = mg(\cos\alpha - \lambda\sin\theta)$ by ❷ and ❸.

Since $R \ge 0$ (for P in contact with plane) for all values of θ, $\cos\alpha \ge \lambda\sin\theta$ as $\theta \to 2\alpha$; that is,

$$\cos\alpha > \lambda 2\sin\alpha\cos\alpha \Rightarrow 1 > 2\lambda\sin\alpha.$$

For **(iii)**, using $F \le \mu R$ (for now) with ❶ and ❸, $mg(\sin\alpha + \lambda\cos\theta) \le \tan\beta\,.mg(\cos\alpha - \lambda\sin\theta)$

$$\Rightarrow \quad \sin\alpha + \lambda\cos\theta \le \frac{\sin\beta}{\cos\beta}(\cos\alpha - \lambda\sin\theta)$$

Solving for λ then leads to $\lambda \le \dfrac{\sin\beta\cos\alpha - \sin\alpha\cos\beta}{\cos\theta\cos\beta + \sin\beta\sin\theta} = \dfrac{\sin(\beta-\alpha)}{\cos(\beta-\theta)}$, as required.

The very final section requires a bit of careful reasoning to arrive at the "corresponding result for $\alpha \le \beta \le 2\alpha$" of $\lambda \le \sin(\beta - \alpha)$.

Question 12

The answer to **(i)** is clearly $\frac{1}{3}(p_1 + p_2 + p_3)$, which turns out to be the p referred to in the rest of the question.

In **(ii)**, and **(iii)** for that matter, there is no harm in setting out the table of outcomes and associated probabilities in tabular form

x	p(x)
2	p^2
1	$2p(1-p)$
0	$(1-p)^2$

and the individual probs. are relatively easy to work out. Then the standard results E(N_1) = $\sum x.\mathrm{p}(x)$ and Var(N_1) = $\sum x^2.\mathrm{p}(x)$ – (E(N_1))^2 give p and $2p(1-p)$ respectively, as required.

If it helps to put your thoughts in order, a tree diagram can help here – remember just to fill in the branches that are relevant – and the probabilities for the various numbers of heads that can arise are

x	p(x)
2	$\frac{1}{3}(p_1p_2 + p_2p_3 + p_3p_1)$
1	$\frac{1}{3}(p_1(1-p_2)+p_1(1-p_3)+p_2(1-p_1)+p_2(1-p_3)+p_3(1-p_1)+p_3(1-p_2))$
0	$\frac{1}{3}((1-p_1)(1-p_2)+(1-p_2)(1-p_3)+(1-p_3)(1-p_1))$

The application of the same standard results then give

$$\mathrm{E}(N_2) = 2p \text{ and } \mathrm{Var}(N_2) = 2p - 4p^2 + \tfrac{2}{3}(p_1p_2 + p_2p_3 + p_3p_1).$$

For **(iv)**, we show that Var(N_1) – Var(N_2) ≥ 0. This boils down to

$$2p^2 - \tfrac{2}{3}(p_1p_2 + p_2p_3 + p_3p_1) \geq 0$$

i.e. $\frac{2}{9}(p_1 + p_2 + p_3)^2 - \frac{2}{3}(p_1p_2 + p_2p_3 + p_3p_1) \geq 0$ or $2(p_1 + p_2 + p_3)^2 - 6(p_1p_2 + p_2p_3 + p_3p_1) \geq 0$ which, upon squaring, gives $2p_1^{\,2} + 2p_2^{\,2} + 2p_3^{\,2} - 2(p_1p_2 + p_2p_3 + p_3p_1) \geq 0$. Now this expression is simply $(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2$ and, being the sum of three squares, is guaranteed to be non-negative. Indeed, it is also immediately clear that equality only occurs when all three squared terms are zero; that is $p_1 = p_2 = p_3$.

Question 13

For Candidate A, if $k \le 2$, she can score a maximum of 4 marks so cannot pass. If $k = 3$, the only way to pass is by getting them all correct, with probability $\frac{1}{n^3}$. For $k = 4$, she can score 5 marks with 3 correct answers and 8 marks with 4 correct answers, giving probability ${}^4C_3 \frac{1}{n^3}(1-\frac{1}{n}) + \frac{1}{n^4} = \frac{4n-3}{n^4}$. Then, finally, if $k = 5$, she can score 7 marks with 4 correct answers and 10 marks with 5 correct answers, so the probability of passing is ${}^5C_4 \frac{1}{n^4}(1-\frac{1}{n}) + \frac{1}{n^5} = \frac{5n-4}{n^5}$. It now remains to demonstrate that $P_4 - P_3 > 0$ and that $P_4 - P_5 > 0$ to justify that $k = 4$ is best.

For Candidate B's strategy, we have a conditional probability:

$$P(k=4 \mid \text{pass}) = \frac{P(k=4 \,\&\, \text{pass})}{P(\text{pass})} = \frac{\frac{1}{6} \times \frac{4n-3}{n^4}}{\left(\frac{1}{6} \times \frac{1}{n^3}\right) + \left(\frac{1}{6} \times \frac{4n-3}{n^4}\right) + \left(\frac{1}{6} \times \frac{5n-4}{n^5}\right)},$$

where the denominator consists of the terms $P(k = 3 \,\&\, \text{pass})$, $P(k = 4 \,\&\, \text{pass})$ and $P(k = 5 \,\&\, \text{pass})$ respectively.

In the case of Candidate C, the probability of passing is just

$$P(3H) \times P(\text{pass} \mid 3H) + P(4H) \times P(\text{pass} \mid 4H) + P(5H) \times P(\text{pass} \mid 5H)$$
$$= {}^5C_3 \frac{n^3}{(n+1)^5} \times \frac{1}{n^3} + {}^5C_4 \frac{n^4}{(n+1)^5} \times \frac{4n-3}{n^4} + {}^5C_5 \frac{n^5}{(n+1)^5} \times \frac{5n-4}{n^5} = \frac{25n-9}{(n+1)^5}.$$